

# Individual Determinacy and Identity Criteria in Ontology-Driven Information Systems

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**Abstract.** The idea that the real-world entities referred to by Information Systems are determinate and uniquely identifiable is a commonly held assumption in the fields of Software Engineering and Database Systems. The concept of identity is also a central topic in Formal Ontology, a discipline that finds application in the field of Information Systems through the use of Foundational Ontologies (FOs). However, while most central concepts of Formal Ontology are, in general explicitly addressed in FOs, the concept of identity has received relatively little attention. The lack of a proper ontological characterization of identity in FOs hinders their application to the analysis of issues related to identification in Information Systems, such as those that arise in conceptual modeling or in database design. This work proposes two distinct, but logically equivalent, formal characterizations of the notion of *individual determinacy*. Moreover, these characterizations are independent of the particularities of a FO's theory and are defined solely in terms of the structure of the FO's models of portions of reality. Finally, it also introduces a few concepts that are useful in the analysis of *identity criteria* for the individuals represented using a FO's theory.

**Keywords.** Formal Ontology, Identification, Information Systems Theory

## Introduction

An information system (IS) can be seen as an “organized combination of people, hardware, software, communications networks, data resources and policies that stores, retrieves, transforms, and disseminates information in an organization” [1]. Differing from systems that directly manipulate real objects, such as manufacturing or logistic systems, ISs manipulate only abstract *references* to real objects. As such, their value depend on the effectiveness of those references: if we cannot determine their referents, the IS's information becomes meaningless. Also, even

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if the determination of referents is a process executed by the IS user, and not by the IS itself, the later still needs to decided whether or not two references have the same referee.

For these reasons, the correct identification of the conditions that rule the determination of identity of an IS's referents is an essential step in its development process. The relevance of those conditions can be evident in the most basic criteria for judging the quality of database tables: for a table to be in first normal form, it must have a key, i.e., there must be a set of attributes that allow the system to determine whether or not two rows refer to the same entity.

An anecdotal case of improper handling of this issue was witnessed by the first author while employed in a judicial court: the table that stored data about human parties in judicial processes lacked a proper key, making it impossible to unequivocally determine the people referred to in its rows due to the existence of homonyms. Needless to say, redundant rows were a frequent issue.

This case illustrates the importance of capturing the identity conditions even before a database is designed. Ideally, the artifact that describes the domain of the IS, the domain's *conceptual model*, should include this information, so that database designers and programmers can design their respective components in a way that ensures the effectiveness of the systems references.

Now, *conceptual modeling* is an activity that has been directly benefited by the theories and research done in the field of Applied Ontology (AO) [2,3]. In particular, this can be seen in the use of *Foundational Ontologies* (FOs) to provide real-world semantics to conceptual modeling languages [4].

Practicing conceptual modeling grounded on a FO means that the conceptual modeler has at his or her disposal a comprehensive collection of general concepts refined by numerous debates in the field of Philosophical Ontology and by their actual application in ISs development practice.

In a FO, one finds formal characterizations for general and domain-independent concepts that are also in the scope of study of the field of Philosophical Ontology, such as *object*, *property*, *parthood*, etc. Since the concept of Identity is a central topic in this field and, as seen above, it is also essential for the development of ISs, it is reasonable to expect that it should receive a comprehensive characterization in FOs. Even though the notion of identity (in multiple guises) plays an important role in ontological analysis [5], in methodologies such as OntoClean [6] or in the characterization of *sortal* universals in FOs [4], a comprehensive analysis about the subject and its relation to representation structures in ISs is still missing in the literature of this area [7,8].

At the root of this problem is the customary way of analyzing the concept of Identity by means of *identity criteria* [9], i.e., predicates that can be used to determine whether or not two references to objects of a certain kind share the same referent. This condition is usually expressed as an axiom analogous to

$$\forall x, y. \text{of-kind}(x, K) \wedge \text{of-kind}(y, K) \longrightarrow (P(x, y) \longleftrightarrow x = y).$$

This strategy presents the following issues: (1) it addresses the concept of Identity solely from a logical perspective; and (2) it admits predicates whose definition provide no real insight regarding the identity conditions of the objects

in its scope. The first issue is because the definition of identity criteria is purely logical and completely neutral with respect to the FO's ontological theory. The other issue is due to the fact that a predicate defined simply as

$$P(x, y) \leftrightarrow x = y.$$

is admissible.

Predicates like these, that rely on logical identity or in the identity of elements that do not represent genuine properties of objects (e.g., *OIDS* - *object identifiers*), cannot be considered *descriptive* accounts of identity criteria, since they provide no genuine information that helps us to understand the identity conditions of the objects analyzed under their scope.

Another closely related issue is that of characterizing the notion of *determinacy*. As mentioned before, a crucial assumption made in the use of ISs is that the objects that the IS refers to can be identified. This assumption is also present in some FOs, which presume that the individuals that compose their models are *determinate*, meaning that the identity of their corresponding objects can be determined. We can characterize *determinacy* by means of special definite descriptions: we call a predicate  $P$  an *individual identity criteria* for  $x$  if and only if it satisfies the following condition:

$$\forall y. P(y) \leftrightarrow y = x.$$

Using this notion, we can say that an individual is *determinate* if there exists an individual identity criteria for  $x$ . This characterization of determinacy, although intuitive, suffers from the same issues pointed up above, i.e., it also admits non-informative predicates as evidence of the determinacy of an individual.

Another important issue is that this characterization does not provide a clear criteria for falsifying the assumption "all individuals are determinate". Although it is usually presumed that real objects are determinate, it does not follow that the corresponding individuals in models of an ontology are also determinate.

As an example, consider an universe consisting in two spheres of steel with the same radius separated in space where one is red and the other is green. The spheres can be identified by their colors and, thus, are determinate. However, a possible conceptualization of this situation might ignore the color of the spheres and only consider their geometrical form, radius and separation in space. In this case, the model of reality would be similar to the Max Black Twin Sphere's example [10]: the representation of the spheres would be indistinguishable but for their separation in space, presenting a symmetry that makes it impossible to define an individual (descriptive) identity criteria for either.

Thus, even though the assumption that objects in real world have a determinate identity may be considered a self-evident fact, the determinacy of an individual in a model of reality admissible by a formal ontological theory is not to be taken for granted. However, if we consider determinacy of an individual as the existence of a predicate in some formalism that serves as an individual identity criteria for it, we arrive at the counter-intuitive situation of having the truthmaker of a property that should depend solely on the ontological facts represented in

the model being dependent also on an arbitrary choice of formal language and of a logical form. Since these choices cannot be explained in terms of the ontological facts represented in the models of the ontology, there is no way to verify or falsify an assumption such as “all individuals are determinate in the model” by analyzing the model itself. In the end, the analysis of identity solely in terms of identity criteria or of individual logical descriptions does not provide enough tools to enable a comprehensive analysis of the identity conditions of individuals.

In this work we address these issues by proposing (1) a characterization of individual identity criteria that avoids non-informative predicates; (2) a logically equivalent characterization of determinacy that is defined in terms of the models themselves and that does not depend on a choice of formal language or of logical forms of predicates; (3) a collection of concepts that can guide the discovery of genuine identity criteria. Furthermore, the theory is presented in a model-independent way, i.e. the characterization is independent of the particular structure of models based on a particular FO. As such, it can be readily applied to any FO that can represent portions of reality.

This paper is organized in 4 sections: we start by presenting a few background concepts in Section 1, followed by the theory, described in Section 2, the description of some applications of the proposed theory in Section 3 and ending with some final considerations in Section 4.

## 1. Background

Foundational Ontologies are systems of general concepts that are applicable across domains, providing a framework that can be used to ground conceptual models, in general, including domain-specific ontologies. FOs such as DOLCE [11], GFO [12], BFO [13] and UFO [4] provide polished and well-tested definitions for general and ubiquitous concepts such as objects, properties, relations, parthood, etc., allowing their users to focus on the concepts that are specific to the domain in consideration while avoiding conceptual pitfalls that have already been addressed by the FO’s theory.

Although all FOs aim to provide a conceptual framework that can be used to construct models of portions of reality, the particularities of each FO’s theory implies that the structure of models of FOs will differ among each other and that the same portion of reality will be represented differently in each FO.

The concepts presented in this work are defined over the structure of models of FOs. However, the theory is presented in a model-independent way, by abstracting the details of each FO’s representation structure in a category-theoretic setting. This strategy has the main benefit of producing a theory that is, in principle, applicable to any FO.

At the core of the theory is the category of *individual structures* of a FO. The objects of this category are the abstract structures that *represent* portions of reality according to the FO’s theory. Those categories are assumed to be *concrete categories*, i.e., their objects are “sets with structure” and their morphisms are functions between the set of individuals of two structures that preserve all the structure properties, i.e., the formal properties and relationships provided by

the FO's theory. As an example of these structures, consider an universe consisting in a wooden table with a green apple on its top. A corresponding individual structure according to, for example, the UFO ontology would be a structure that includes elements representing the table and the apple, called *substantials*, and elements representing particularized properties of those substantials, called *moments*, e.g., the table's capacity of sustaining the apple's weight, the apple's weight and greenness or the spatial relation between the table and the apple.

The concept of Determinacy proposed in this work is characterized in terms of the non-existence of certain types of morphisms in the category of individual structures. This dualism between a logical notion of identity criteria and the structural properties of a model was explored in detail in [14], where the author shows the relationship between grades of discrimination and grades of symmetry in the context of first-order logic theories and first-order structures.

## 2. Contribution

### 2.1. Individual Determinacy

The objects human beings refer to as particulars are generally considered to be determinate, at least on a mesoscopic level. The fact that we can recognize an object among the other objects in the universe means that this object has a sufficiently number of properties and relationships that allows its identity to be determined.

An object's identifiability can be characterized as the existence of a predicate that is satisfied exactly by the element that represents the object in an individual structure and no other. For example, supposing that fingerprint patterns are unique properties of human beings, we might say that human beings are identifiable because it is possible to define, for each one, a definite description predicate that picks it by comparing fingerprints.

The characterization of the notion of Identity or Determinacy in FOs, when present, usually follows a similar strategy, i.e., by requiring the existence of suitably defined predicate, called an *identity criteria*, that agrees with logical equality when applied to the elements of an individual structure. This strategy, however, has the undesirable effect of introducing a dependency from an ontological concept (determinacy) to a logical concept (definability of a predicate). This strategy, hereby called the *logical characterization of determinacy*, also presents other difficulties:

- predicates can satisfy the requirement of agreeing with logical identity while failing to have any informational value, such as when a trivial definition is used, as in

$$P(x, y) \longleftrightarrow x = y. \tag{1}$$

- a particular choice of formal language and of a logical form for the predicate cannot be justified ontologically, since this variables (formal language, logical form) represent epistemic or logical choices.

The following definitions provide an alternate characterization for the notion of determinacy, as a function of the set of isomorphisms of an individual structure. We say that an individual is determinate in an individual structure if it is impossible to exchange it with another individual without invalidating some fact about the individuals in the structure. Conversely, we consider an individual indeterminate if there is a way to permute it with another individual in such a way that no change can be perceived in the individual structure:

**Definition 1** (Permutability). *An individual  $x$  of an individual structure  $S$  is said to be permutable with another individual  $y$  if and only if there is a permutation of  $S$ , represented by an isomorphism of  $S$  into itself, that maps  $x$  to  $y$ .*

**Definition 2** (Individual determinacy). *An individual  $x$  of a individual structure  $S$  is considered determinate if and only if all permutations of  $S$  preserve  $x$ , i.e.  $x$  cannot be swapped seamlessly with a distinct individual of  $S$ .*

Using this concept, it is possible to formalize the assumption that a class of individuals of a FO is determinate, by requiring that admissible individual structures have no non-trivial permutations of individuals in this class, ruling out ambiguous structures, such as the twin spheres structure presented in [10].

## 2.2. Individual Identity Criteria

The concept of *identity criteria*, also referred to as *identity condition*, is frequently used to characterize the notion of identity of objects of a certain kind. By focusing in a single individual instead of a class, we can define the notion of an *individual identity criteria* of an individual  $x$  as a special definite description of  $x$  that singles out  $x$  in a particular structure, i.e., a predicate that is only satisfiable by  $x$  in that structure. The identity criteria that applies to all instances of a class can be seen as capturing a common pattern of individual identity criteria for its instances.

A tentative definition for an individual identity criteria for  $x$  in a structure  $S$  could define it as a predicate  $P$  that singles-out  $x$  in  $S$ :

$$\text{id-criteria}^*(P, x, S) \longleftrightarrow \forall y \in S. P(y) \longrightarrow y = x \quad (2)$$

One such predicate would be, for example,

$$P(x) \longleftrightarrow \text{car}(x) \wedge \text{brazillian-license-plate}(x, \text{JFW-2427}) \quad (3)$$

which picks up exactly one of the author's previous vehicles. Another logically equivalent predicate would be:

$$P(x) \longleftrightarrow x = \text{AuthorsOldCar} \quad (4)$$

where `AuthorsOldCar` is the label that refers to the same car in the structure.

Although predicates satisfy the condition expressed in (2), the last one provides no useful information about what makes the individual unique. Instead, it relies on a particular labeling of objects in the structure, which does not necessarily have any ontological implications.

An effective individual identity criteria should not depend on any non-ontological features of an individual structure. This idea can be characterized by requiring that a suitable predicate be *invariant* under individual structure morphisms:

$$\text{id-criteria}(P, x, S) \longleftrightarrow \forall S'. \forall \varphi \in \mathbf{Iso}(S, S'). \forall y \in S'. P(S', y) \longrightarrow y = \varphi(x) \quad (5)$$

Which predicates satisfy this condition depend on what is considered non-ontological in the FO's category of individual structures. In the previous example, if labels assigned to objects are considered arbitrary and ontologically irrelevant while license-plate numbers are considered ontological meaningful, predicate (4) would be rejected while (3) would be admissible. Thus, this characterization also helps ensure that the predicates will be informative, by disallowing "shortcuts" that rely inappropriately on logical identity and forcing the predicate to actually describe the individual using the ontological elements represented in the structure.

Although the concept of individual identity criteria is defined by means of the existence of a predicate and the notion of individual determinacy is defined solely in terms of properties of the category of individual structures, the two concepts are closely related, as can be shown in the following theorems:

**Theorem 1** (Individual identity criteria existence implies determinacy). *Given a individual structure  $S$  and an individual  $x$  of  $S$ , if there is a predicate  $P$  that satisfies the conditions in (5) for an individual identity criteria for  $x$  in  $S$ , then the individual  $x$  is determinate in  $x$ , as per (2).*

*Proof.* Suppose there is a predicate  $P$  that attends the conditions for being an individual criteria for  $\mathbf{x}$  in  $S$ . By the definition of individual identity criteria (5), we have that

$$\forall S'. \forall \varphi \in \mathbf{Iso}(S, S'). \forall y. P(S', y) \longrightarrow y = \varphi(\mathbf{x}) \quad (6)$$

The identity morphism in  $S$  is also an isomorphism. By taking  $S' = S$  and using the identity morphism in (6), we have

$$\forall y. P(S, y) \longrightarrow y = \mathbf{x} \quad (7)$$

and, thus, that

$$P(S, \mathbf{s}) \quad (8)$$

which is expected, since  $P$  is an identity criteria for  $\mathbf{s}$  in  $S$ .

Now, suppose  $\varphi^*$  is an arbitrary permutation of  $S$ . By the invariance of  $P$  (6), we would have

$$\forall y. P(S, y) \longrightarrow y = \varphi^*(\mathbf{x}) \quad (9)$$

thus, by taking  $y = \mathbf{x}$  and using (7) we have that:

$$\mathbf{x} = \varphi^*(\mathbf{x}) \quad (10)$$

and, consequently, that any permutation of  $S$  maps  $\mathbf{x}$  to itself. Thus,  $\mathbf{x}$  is a determinate individual in  $S$ .  $\square$

**Lemma 1** (determinacy implies invariance). *For any individual structure  $S$ , any individual  $\mathbf{x}$  of  $S$ , and isomorphisms  $\varphi_1$  and  $\varphi_2$  between  $S$  and another structure  $S'$ , if  $\mathbf{x}$  is determinate in  $S$  then  $\varphi_1$  and  $\varphi_2$  agree on the image of  $\mathbf{x}$ , i.e.  $\varphi_1(\mathbf{x}) = \varphi_2(\mathbf{x})$ .*

*Proof.* Suppose, hypothetically, that there exist isomorphisms  $\varphi_1$  and  $\varphi_2$  between  $S$  and  $S'$  such that

$$\varphi_1(\mathbf{x}) \neq \varphi_2(\mathbf{x}) \quad (11)$$

Since  $\varphi_1$  and  $\varphi_2$  are isomorphisms, they both have inverses, resp.  $\varphi_1^-$  and  $\varphi_2^-$ , and those are also isomorphisms. Thus,  $\varphi_2^- \circ \varphi_1$  is also an isomorphism and, thus, a permutation of  $S$ . Since, by hypothesis, all permutations of  $S$  map  $\mathbf{x}$  to itself, we have that

$$\varphi_2^-(\varphi_1(\mathbf{x})) = \mathbf{x} \quad (12)$$

We also have that  $\varphi_2^- \circ \varphi_2$  is another permutation of  $S$ , and, similarly, that

$$\varphi_2^-(\varphi_2(\mathbf{x})) = \mathbf{x} \quad (13)$$

and, thus, that

$$\varphi_2^-(\varphi_1(\mathbf{x})) = \varphi_2^-(\varphi_2(\mathbf{x})) \quad (14)$$

However, since  $\varphi_2^-$  is injective, we have that  $\varphi_1(\mathbf{x}) = \varphi_2(\mathbf{x})$ , contradicting our hypothesis and thus we can conclude, by discharging the hypothesis on  $\varphi_1$  and  $\varphi_2$ , that all isomorphisms between  $S$  and  $S'$  agree on the target of  $\mathbf{x}$ :

$$\forall S'. \forall \varphi_1, \varphi_2 \in \mathbf{Iso}(S, S'). \varphi_1(\mathbf{x}) = \varphi_2(\mathbf{x}) \quad (15)$$

$\square$

The next theorem demonstrates that the determinacy of an individual in an individual structure is a sufficient condition for the (implicit) definability of an identity criteria in a logic that allows higher-order quantification and that has the Hilbert's choice operator:

**Theorem 2** (Determinacy implies individual identity criteria definability). *For every individual structure  $S$  and every individual  $\mathbf{x}$  of  $S$ , if  $\mathbf{x}$  is determinate in  $S$  then it is possible to define, using the Hilbert's choice operator, a predicate that satisfies the conditions for an individual identity criteria for  $\mathbf{x}$  in  $S$ .*



*Proof.* Using the syntax for the Hilbert's choice operator used in the Isabelle/HOL language,  $(\text{SOME } x. P x)$ , which denotes some object that satisfies the predicate  $P$ , we can define a tentative individual identity criteria for  $\mathbf{x}$  in  $S$  as the following predicate  $P$ :

$$\forall S'. \forall y \in S'. P(S', y) \longleftrightarrow (\text{SOME } \varphi. \varphi \in \text{Iso}(S, S'))(\mathbf{x}) = y \quad (16)$$

In order to show that  $P$  is actually an individual identity criteria for  $\mathbf{x}$  in  $S$ , we need to prove that  $P$  is invariant under individual structure isomorphisms, i.e. that for any isomorphism  $\varphi^*$  between  $S$  and an individual structure  $S'$ ,  $P$  picks exactly the image of  $\mathbf{x}$  under  $\varphi^*$ :

$$\forall y \in S'. P(S', y) \longrightarrow y = \varphi^*(\mathbf{x})$$

Now, supposing  $y$  is an arbitrary individual in  $S$ , we have that

$$P(S', y) \longleftrightarrow (\text{SOME } \varphi. \varphi \in \text{Iso}(S, S'))(\mathbf{x}) = y \quad (17)$$

But, according to Lemma 1, any two isomorphisms between  $S$  and  $S'$  agree on the image of  $\mathbf{x}$ . In particular, we have that

$$\forall \varphi \in \mathbf{Iso}(S, S'). \varphi(\mathbf{x}) = \varphi^*(\mathbf{x}) \quad (18)$$

Thus, whatever choice we make for  $\varphi$  in (17), we would have  $\varphi$  having the same image as  $\varphi^*$ . Further, we have that

$$P(S', y) \longleftrightarrow \varphi^*(\mathbf{x}) = y \quad (19)$$

as we aimed to demonstrate, proving that an individual identity criteria for  $\mathbf{x}$  is definable in  $S$ .  $\square$

Note that the definition of an individual identity criteria using the Hilbert's operator is an indirect definition, which is by itself not informative regarding what makes  $\mathbf{x}$  unique in the structure, but such information can be extracted from the proof that the individual  $\mathbf{x}$  is determinate. Note also that the existence of this indirect definition does not imply that a direct and informative definition can be expressed in a particular formalism, or in any formalism at all, since the necessary conditions for identifying  $x$  might not be finitely axiomatizable in a particular logical formalism.

Nevertheless, the relationship between the notion of individual determinacy and of identity criteria hints that the analysis of the ontological characteristics that allows an informative identity criteria for  $\mathbf{x}$  to be expressed in the context of an individual structure  $S$  can be found by understanding what makes  $\mathbf{x}$  non-permutable in  $S$ .

### 2.3. Anchors and Determination Contexts

The characterization of determinacy presented in Section 2.1 is expressed by means of a non-existence condition, i.e., non-existence of permutations of  $x$ . This definition is useful in that it expresses the intuition that an individual is determinate if it cannot be exchanged seamlessly with another.

However, this definition does explain exactly what elements in the structure ensure the uniqueness of an individual, i.e. it does not explain what are the truthmakers that ground the determinacy of an individual. This section presents an alternate but logically equivalent characterization that is existential in form, in that it expresses determinacy as the *existence* of certain substructures of an individual structure that ground the determinacy of a certain individual. These substructures are called *determination contexts* and they are characterized by the existence of certain morphisms, called *anchors*.

#### 2.3.1. Anchors and determinacy

**Definition 3** (Anchor). *If  $S$  and  $S'$  are configurations of individuals,  $x$  is an individual of  $S$  and  $y$  is an individual of  $S'$ , with an injective morphism  $\varphi$  from  $S$  to  $S'$  that maps  $x$  to  $y$ , such that all injective morphisms from  $S$  and  $S'$  also map  $x$  to  $y$ , we say that  $\varphi$  is an anchor for  $y$  in  $S'$ , and that  $S$  is an anchoring structure for  $y$  in  $S'$ .*

The relation between the existence of an *anchoring* for an individual  $x$  in a configuration  $S$  and  $x$ 's determinacy in  $S$  is given by the following theorems:

**Theorem 3** (Determinacy implies anchoring is possible). *If an individual  $x$  is determinate in a individual structure  $S$ , then there is at least one anchoring morphism for  $x$  in  $S$ .*

*Proof.* Since any injection from  $S$  to  $S$  itself is necessarily a surjection, and thus a permutation of  $S$ , and since all permutations of  $S$  map  $x$  to itself, because  $x$  is determinate in  $S$ , then the identity morphism in  $S$  is an anchor for  $x$  in  $S$ .  $\square$

**Theorem 4** (Anchoring implies determinacy). *If an individual  $x$  has an anchor  $\varphi$  in  $S$ , then  $x$  is determinate in  $S$ .*

*Proof.* Suppose, by hypothesis, that  $x$  has an anchor  $\varphi$  in  $S$ , that maps  $y$  in  $S'$  to  $x$  in  $S$ . Suppose also that  $x$  is not determinate in  $S$ . We would have, then, that

$$\forall \varphi^* \in \text{InjMorph}(S', S). \varphi^*(y) = x \wedge (\forall z \in S'. \varphi(z) = x \longrightarrow z = y) \quad (20)$$

$$\exists \varphi_1, \varphi_2 \in \text{Perm}(S). \varphi_1(x) \neq \varphi_2(x) \quad (21)$$

Now, since  $\varphi_1$  and  $\varphi_2$  are permutations of  $S$  and  $\varphi$  is an injection from  $S'$  to  $S$ , then the following are also injections from  $S'$  to  $S$ :

$$\varphi_1^* \equiv \varphi_1 \circ \varphi \quad (22)$$

$$\varphi_2^* \equiv \varphi_2 \circ \varphi \quad (23)$$

We have, by hypothesis (21), that  $\varphi_1(x) \neq \varphi_2(x)$  and, thus, that

$$\begin{aligned}\varphi_1^*(x) &= (\varphi_1 \circ \varphi)(x) \\ &\neq (\varphi_2 \circ \varphi)(x) \\ &= \varphi_2^*(x)\end{aligned}$$

and thus, that

$$\varphi_1^*(x) \neq \varphi_2^*(x) \quad (24)$$

However, both  $\varphi_1^*$   $\varphi_2^*$  are injections from  $S'$  to  $S$  and, thus we have by (20) that

$$\varphi_1^*(x) = \varphi_2^*(x) \quad (25)$$

contradicting (24).

We can conclude then that  $x$  must be determinate in  $S$ .

□

### 2.3.2. Determination Contexts

**Definition 4** (Substructure). *We say that a individual structure  $S'$  is a substructure of  $S$  if and only if there is an inclusion morphism  $\varphi$  between  $S$  and  $S'$ , where a morphism is considered an inclusion morphism if it is a morphism between a configuration and another configuration whose set of individuals is a superset of the set of individuals of the former.*

**Definition 5** (Determination Context). *We call an anchoring structure for  $x$  in  $S$  that is also a substructure of  $S$  a determination context for  $x$ .*

Since the existence of an anchoring structure for  $x$  implies the existence of a determination context and since a determination context is itself an anchoring structure for  $x$ , the existence of a determination context for  $x$  in  $S$  is equivalent to the existence of an anchor for  $x$  in  $S$  and thus, by Theorem (4), the existence of determination contexts for  $x$  in  $S$  is logically equivalent to the determinacy of  $x$  in  $S$ .

For example, if either the DNA code or the fingerprint of a person  $x$  is sufficient to determine the identity of  $x$  in a context represented by a configuration  $S$ , there would exist at least two determination contexts for  $x$ , one consisting in  $x$  with its DNA code and another consisting in  $x$  with its fingerprint.

**Definition 6** (Minimal Determination Context). *A determination context  $S'$  for  $x$  is considered minimal if no other context for  $x$  is also a proper substructure of  $S'$ .*

Minimal determination contexts represent minimal sets of sufficient properties for the determination of  $x$ 's identity in  $S$ .

Using the notion of determination context, we can define the notions of weak and strong identification dependency between individuals in a configuration:

**Definition 7** (Weak Identification Dependency). *We say that an individual  $x$  of  $S$  is weakly identification-dependent upon an individual  $y$  of  $S$  if and only if there exists a minimal determination context for  $x$  that contains  $y$*

**Definition 8** (Strong Identification Dependency). *We say that an individual  $x$  of  $S$  is strongly identification-dependent upon an individual  $y$  of  $S$  if and only if  $x$  is determinate in  $S$  and all determination contexts of  $x$  also includes  $y$ .*

The relation of weak identification dependency can be used to identify possible elements for use in the definition of identity criteria, while the relation of strong identification dependency indicate elements that are necessary for the definition of an identity criteria of an individual, i.e. that must be included in any definition of an identity criteria for that individual.

The notion of strong identification-dependency is also transitive and symmetric, configuring a pre-order over determinate individuals of a configuration. This relation can be used to determine the set of individuals that are essential for the identification of the individuals in the configuration. This set can serve as a basis for producing identity criteria.

The concept of *determination contexts* can also be used to determine whether the identity of an individual is an intrinsic or an extrinsic property of that individual. An individual can be considered *intrinsically determinate* if there is at least one determination context for it that only includes itself (or only itself and its parts, moments, etc.). Conversely, it can be considered *extrinsically determinate* if all of its determination contexts contain other *externally dependent* object.

For example, consider a configuration of solid objects in a three-dimensional vacuum, where the objects are all intrinsically symmetric, i.e., they have exactly the same intrinsic properties. Suppose that the configuration includes the relationship of distance between these solid objects. In this configuration, the minimal identification contexts would be composed by sets of objects related by the distance relation to at most 4 other objects, since in three dimensions the relative position of an object can be determined by its distance to other 4 non-coplanar points.

### 3. Applications

As mentioned in the introduction of this paper, ISs are designed with the assumption that the identification of objects they refer to can be determined, not only in reality but also through the data available in the IS's abstraction of that portion of reality.

At a mesoscopic level, it is a commonly held assumption that the identity of individuals in reality is determinate. However, it might be the case that a representation of that reality embedded in an IS does not have enough details to determine the identity of the referred objects. The concept of *determinacy* presented in this work presents an objective criteria to validate the assumption that the representation embedded in an IS is sufficiently rich to enable the identity of its represented objects to be determined.

This has a direct application during conceptual modeling: while investigating the domain and gathering examples of objects, properties and relationships, the conceptual modeler can validate this assumption by looking for non-trivial permutations in its sample data. With the assistance of a software tool, this validation can be continuously supported, in which case the tool could provide the conceptual modeler with proofs that the identity of the samples is determinate, in the form of *determination contexts*, or with a report about which objects are still indeterminate in the sample and need further investigation.

Besides serving as a methodological tool, the concepts presented in this work (e.g. *minimal determination contexts*, *identification dependency*, etc.), when considered in the context of a particular FO, can help in the analysis of issues related to identity when using that FO as an ontological foundation.

#### 4. Final Considerations

This work presents a formal theory characterizing the notion of Individual Determinacy and its relation to the notion of Individual Identity Criteria.

This theory considers the determinacy of an individual in a representation of a portion of reality as its non-permutability with other individuals in that representation. In particular, it advances a pair of theorems that demonstrates a close relationship between the notion of individual identity criteria and that of individual determinacy. The theory presents also an alternate, but logically equivalent, definition for the notion of determinacy of an individual expressed as the existence of a determination context for that individual. This notion, in turn, plays two useful roles: it serves as truthmaker for the assertion that an individual is determinate; and it puts in evidence the elements of the individual structure that can be used to define identification criteria for that individual. Finally, the theory also introduces several concepts that should be useful in the analysis of questions related to identity in the context of an FO, such as the concept of minimal identification contexts and of strong and weak identification dependency.

In a future work, we expect that the presented characterization of determinacy will allow us to draw an explicit connection with an alternative formal elaboration of the notion of *Sortals* in FO's. From a methodological perspective, this shall allow us to develop precise support for identifying those universals that, given a conceptualization of a domain, supplies identity criteria for the individuals in that domain, i.e., the so-called *Ultimate Sortals* [4].

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